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Unified International Mathematics Olympiad

UNIFIED INTERNATIONAL MATHEMATICS OLYMPIAD (UPDATED)

CLASS - 9
Question Paper Code : 40119

KEY

1	2	3	4	5	6	7	8	9	10
D	A	C	A	Delete	D	A	B	C	D
11	**12**	**13**	**14**	**15**	**16**	**17**	**18**	**19**	**20**
A	C	A	B	C	B	B	C	A	Delete
21	**22**	**23**	**24**	**25**	**26**	**27**	**28**	**29**	**30**
D	D	B	C	B	D	C	A	A	D
31	**32**	**33**	**34**	**35**	**36**	**37**	**38**	**39**	**40**
A,B,C,D	A,B,C	A,B,C,D	A,B,C,D	A,C,D	A	A	C	B	A
41	**42**	**43**	**44**	**45**	**46**	**47**	**48**	**49**	**50**
C	D	D	D	C	B	A	D	D	A

EXPLANATIONS

MATHEMATICS

01. (D) $m - \not{n} + m + \not{n} = 4 + 20$

$2m = 24 \Rightarrow m = \frac{24}{2} = 12$

02. (A) Let $\sqrt[4]{124 + 32\sqrt{15}} = \sqrt{\sqrt{a} + \sqrt{b}}$

Square twice to get

a + b = 124 and ab = 3840

a = 64, b = 60

Simplify $\sqrt{8 + 2\sqrt{15}} = \sqrt{5} + \sqrt{3}$

03. (C) Given $p(x) = 2x^3 - 11x^2 + 17x - 6$

$p(-1) = 2(-1)^3 - 11(-1)^2 + 17(-1) - 6 = -2 - 11 - 17 - 6$

$p(-1) \neq 0 \Rightarrow (x + 1)$ is not a factor of $p(x)$

$p(2) = 2(2)^3 - 11(2)^2 + 17(2) - 6$

$= 16 - 44 + 34 - 6$

$p(2) = 0 \Rightarrow (x - 2)$ is a factor of $p(x)$

04. (A) $x = 40° + 65° = 105°$

05. Delete

06. (D) Area of $\Delta AOB = \frac{1}{4}$ area of the 11 gm ABCD

$= \frac{1}{4} \times 52\ cm^2$

$= 13\ cm^2$

07. (A) Volume = $\frac{4}{3}\pi r^3 = 38808$

$\frac{4}{3} \times \frac{22}{7} \times r^3 = 38{,}808\ cm^3$

$r^3 = 38808 \times \frac{3 \times 7}{4 \times 22}$

$r = \sqrt[3]{21^2 \times 21}$

r = 21 cm

Surface area of a sphere

$= 4\pi r^2 = 4 \times \frac{22}{7} \times 21 \times 21$

$= 5544\ cm^2$

08. (B) $\angle CBA = 180° - 100° = 80°$

But $\angle CBA + \angle ADC = 180°$

$80° + \angle ADC = 180°$

$\angle ADC = 100°$

But $\angle CDF + \angle ADC = 180°$

$\angle CDF + 100° = 180°$

$\angle CDF = 80°$

09. (C) $p(1) = (1)^3 - (1)^2 + 1 + 1 = 1 - 1 + 1 + 1 = 2$

$p(-1) = (-1)^3 - (-1)^2 + (-1) + 1 = -1 - 1 - 1 + 1 = 2$

$\frac{p(1) + p(-1)}{2} = \frac{2 + (-2)}{2} = 0$

10. (D) $5y + y + 2y + y = 180°$ [$\because$ AB || CD]

$9y = 180°$

$y = \frac{180°}{9} \Rightarrow y = 20°$

11. (A) From the second equation

$x = y - 1$

Substitute into the first equation

$3(y - 1) - 5y = -1$

$3y - 3 - 5y = -1$

$-2y = 2$

$y = -1$

Then, $x = -1 - 1 = -2$

12. (C) A linear equation in two variables always has infinite solutions (line).

13. (A) Given AB = AC $\Rightarrow$ ACB = ABC = x

In BOC, given OB = OC OCB = OBC = y

But ABO + OBC = x

ABO + $y = xy$

ABO = $x - y$

similarly we can prove ACO = $x - y$

$\therefore$ ABO : ACO = $(x - y) : (x - y) = 1 : 1$

14. (B) The cube's space diagonal $\left(a\sqrt{3}\right)$ equals the sphere's diameter $\left(3\sqrt{3}\ cm\right)$

Solve for side length a = 3 cm.

Volume = $a^3 = 27\ cm^3$

15. (C) ABCD is a cyclic quadrilateral.

$\therefore$ $\angle A + \angle C = 180°$

$\angle A + 100° = 180°$

$\angle A = 80°$

In ΔABD, $80° + 30° + \angle ADB = 180°$

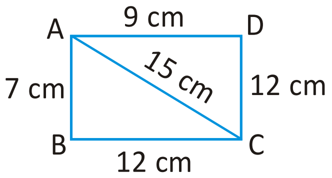
$\angle ADB = 180° - 110° = 70°$

16. (B) Volume of the pyramid

$= \frac{1}{3} \times \text{base area} \times \text{height}$

$= \frac{1}{3} \times 8 \times 8 \times 12\ cm^3 = 256\ cm^3$

17. (B) Area of $\Delta ABC = \sqrt{s(s-a)(s-b)(s-c)}$



where $s = \frac{a+b+c}{2} = \frac{7cm+12cm+15cm}{2}$

$= \frac{34cm}{2} = 17$ cm

Area $(\Delta ABC) = \sqrt{17\times10\times5\times2}$ cm^2

$= 10\sqrt{17}$ cm^2

In ΔACD, $15^2 = 12^2 + 9^2 \Rightarrow \angle D = 90°$

$\therefore$ Area of $\Delta ACD = \frac{1}{2} \times AD \times DC$

$= \frac{1}{\cancel{2}_1} \times 9 \times \cancel{12}^6$ cm^2 = 54 cm^2

$\therefore$ Area of the quadrilateral ABCD

= $(10\sqrt{7} + 54)$ cm^2

18. (C) Given $p(x) = \frac{2x+1}{x-2}$

$$\therefore \quad p\left(\frac{2x+1}{x-2}\right) = \frac{2\left(\frac{2x+1}{x-2}\right)+1}{\left(\frac{2x+1}{x-2}\right)-2}$$

$$= \frac{\left[\frac{4x+2+(x-2)}{x-2}\right]}{\left[\frac{2x+1-2(x-2)}{x-2}\right]}$$

$$= \frac{(4x+\cancel{2}+x-\cancel{2})}{\cancel{(x-2)}} \times \frac{\cancel{(x-2)}}{(\cancel{2x}+1-\cancel{2x}+4)}$$

$$= \frac{5x}{5} = x$$

19. (A) If $x = 2 + 2^{\frac{1}{3}} + 2^{\frac{2}{3}}$

$\Rightarrow x - 2 = 2^{\frac{1}{3}} + 2^{\frac{2}{3}}$

Cubing on both sides, we get

$(x-2)^3 = \left(2^{\frac{1}{3}} + 2^{\frac{2}{3}}\right)^3$

$\Rightarrow x^3 - 6x(x-2) - 8 = 2 + 6(x-2) + 4$

$\Rightarrow x^3 - 6x^2 + 12x - 8 = 6 + 6(x-2)$

$\Rightarrow x^3 - 6x^2 + 6x = -6 + 8$

$\Rightarrow x^3 - 6x^2 + 6x = 2$

20. Delete

21. (D) $(x^4 - x^3 + x - 1) = x^3(x-1) + 1(x-1)$

$= (x-1)(x^3+1)$

$= (x-1)(x+1)(x^2 - x + 1)$

$x^4 + x^2 + 1 = x^4 + x^2 + 1 + x^2 - x^2$

$= (x^4 + 2x^2 + 1) - x^2$

$= (x^2+1)^2 - x^2$

$= (x^2 + x + 1)(x^2 - x + 1)$

$\therefore$ HCF $= (x^2 - x + 1)$

22. (D) Given a = 11 cm

b = 60 cm, c = 61 cm

$\therefore s = \frac{a+b+c}{2}$

$= \frac{(11+60+61)\text{ cm}}{2} = \frac{132\text{ cm}}{2} = 66$ cm

Area of ΔABC

$= \sqrt{66\times(66-11)(66-60)(66-61)}$ cm^2

$= \sqrt{66\times55\times6\times5}$ cm^2

= 11 × 6 × 5 cm^2 = 330 cm^2

$\frac{1}{2}$ × biggest side × shortest altitude

= 330 cm^2

$\frac{1}{2}$ × 61 cm × shortest altitude = 330 cm2

$\therefore$ Shortest altitude = 330 cm^2 × $\frac{2}{61}$ cm

$= 10.819\ cm^2 = 10.82\ cm^2$

23. (B) $\angle PTQ = 90°$ [$\because$ Angle in a semicircle]

TQRS is a cyclic quadrilateral

$\therefore$ $\angle QTS + \angle SRQ = 180°$

$\therefore$ $x + y = \angle PTQ + \angle QTS + \angle SRQ = 90° + 180° = 270°$

24. (C) (0,0), (–1,1), (1,1), (0,2) are four ordered pairs satisfies the given condition that $x^{2020} + y^2 = 2y$

25. (B) Let the height be 'x'

$\therefore \text{Radius} = 1\frac{2}{3}x = \frac{5}{3}x$

Given $2\pi rh = 4620\ cm^2$

$\Rightarrow 2 \times \frac{22}{7} \times \frac{5x}{3} \times x = 4620\ cm^2$

$x^2 = \cancel{4620}^{\cancel{210}^{\cancel{105}^{21}}}\ cm^2 \times \frac{1}{\cancel{2}} \times \frac{7}{\cancel{22}_1} \times \frac{3}{\cancel{5}_1}$

$x^2 = (21cm)^2$

$\therefore x = 21$ cm

$\therefore \text{Radius} = \frac{5}{3}x = \frac{5 \times \cancel{21}^7 cm}{\cancel{3}_1} = 35cm$

Total surface area = $2\pi r\ (h + r)$

$= 2 \times \frac{22}{\cancel{7}_1} \times \cancel{35}^5 cm(21 + 35)cm$

= 220 cm × 56cm

$= 12320\ cm^2$

26. (D) Area of the sector = CSA of a cone

$\frac{1}{2}\cancel{\pi}(2\sqrt{3}\ cm)^2 = \cancel{\pi}r \times (2\sqrt{3})$

$\frac{1}{2}\frac{(2\sqrt{3})^{\cancel{2}}}{\cancel{(2\sqrt{3})}} = r$

$r = \sqrt{3}$

$h = \sqrt{l^2 - r^2} = \sqrt{(2\sqrt{3})^2 - \sqrt{3}^2}$

$= \sqrt{12 - 3} = \sqrt{9} = 3$

Voluem of the cone = $\frac{1}{3}\pi r^2 h$

$= \frac{1}{\cancel{3}}\pi \times (\sqrt{3})^2 \times \cancel{3} = 3\pi\ cm^3$

27. (C) Maximum length of pencil = diagonal of a cubord

$= \sqrt{l^2 + b^2 + h^2}$

$= \sqrt{(12)^2 + (9)^2 + 8^2}$ cm

$= \sqrt{144 + 81 + 64}$ cm

$= \sqrt{289}$ cm

= 17 cm

28. (A) LHS = $\cancel{a^3} - \cancel{b^3} + \cancel{b^3} - \cancel{c^3} + \cancel{c^3} - \cancel{a^3} = 0$

29. (A) $\left(\sqrt[3]{x} + \sqrt[3]{y}\right)\left(\sqrt[3]{x^2} - \sqrt[3]{xy} + \sqrt[3]{y^2}\right)$

$= \left(\sqrt[3]{x}\right)^3 + \left(\sqrt[3]{y}\right)^3$

$= (x + y)$

30. (D) Given angle = 45° $\Rightarrow$ Height of empty part = Diameter = 6 cm

$\therefore$ Volume of the remaining water

$= \pi \times 3 \times 3 \times 11\ cm^3 - \frac{1}{\cancel{2}} \times \pi \times 3 \times 3 \times \cancel{6}^3\ cm^3$

$= 99\pi\ cm^3 - 27\pi\ cm^3$

$= 72\pi\ cm^3$

$= 72\pi$ ml [$\because 1\ cm^3 = 1\ ml$]

MATHEMATICS - 2

31. (A,B,C,D)

Bring everything to one side:

$a^2 + b^2 + c^2 - ab - bc - ca = 0$

This is equivalent to:

$$\frac{1}{2}[(a-b)^2 + (b-c)^2 + (c-a)^2] = 0$$

This implies:

$(a-b)^2 = 0, (b-c)^2 = 0, (c-a)^2 = 0$

$a = b , b = c , c = a$

$\Rightarrow a = b = c$

32. (A,B,C)

In a rhombus, kite and square the diagonal are always perpendicular.

33. (A,B,C,D)

$4(0) + 5(2.8) = 14$

$\therefore$ (0, 2.8) lies on $4x + 5y = 14$

$4(3) + 5(0.4) = 12 + 2 = 14$

$\therefore$ (3, 0.4) lies in $4x + 5y = 14$

$4(-1.5) + 5(4) = -6 + 20 = 14$

$\therefore$ (–1.5, 4) lies on $4x + 5y = 14$

$4(5) + 5(-1.2) = 20 - 6 = 14$

$\therefore$ (5, –1.2) lies on $4x + 5y = 14$

34. (A,B,C,D)

Option (B) : $0 \times \sqrt{5} = 0$

The product of a rational number and an irrational number is some times rational number and some times irrational number.

$\therefore$ The product of a rational number and an irrational number need not be an irrational number.

All options are true.

35. (A,C,D)

$\angle A = \angle B + 15°$

$\angle C = \angle B - 30°$

But $\angle A + \angle B + \angle C = 180°$

$\angle B + 15° + \angle B + \angle B - 30° = 180°$

$3\angle B - 15° = 180°$

$3\angle B = 195°$

$$\angle B = \frac{195°}{3} = 65°$$

$\angle A = 65° + 15° = 80°$

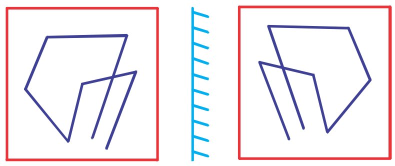
$\angle C = 65° - 30° = 35°$

$\angle A + \angle B = 80° + 65° = 145°$

REASONING

36. (A) R is the sister of P, which means R and P are siblings. P is the son of Q, so Q is the parent of P, and therefore also the parent of R. T is given as the mother of Q. Since Q is R's parent, T is R's grandparent. Specifically, as T is the mother of R's parent, R is T's granddaughter.

37. (A)



38. (C) One of the dots lies in the region common to the circle the triangle and the rectangle only, another dot lies in the region common to the circle and the triangle only and the third dot lies in the region common to the circle, the square and the triangle only. In each of the figures (1), (2) and (4) there is no region common to the circle, the triangle and the rectangle only. Only fig. (3) consists of all the three types of regions.

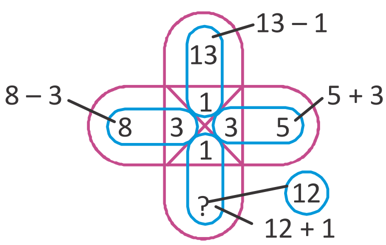
39. (B) Initially facing East

90° clockwise $\rightarrow$ South

45° anti-clockwise $\rightarrow$ halfway between East and South = South - East

40. (A) If in the word a letter is oth letter from the beginning of the english alphabet then in the code, the corresponding letter is the (n + 1)th letter from the end of the alphabet.

41. (C)



42. (D) From both the figures we find that numbers 1,2 and 4 dots appear adjacent to 6. thus, the number 5 dots will appear opposite to 6. Therefore when 6 is at the bottom, then 5 will be at the top.

43. (D) (A) 8 – 43 ÷ 6 × 1 = 4 (✗)

(B) 7 × 6 ÷ 2 + 3 – 6 = 5 (✗)

(C) 10 + 7 – 3 ÷ 2 = 4 (✗)

(D) 2 × 5 – 6 + 2 = 6 (✓)

44. (D) The arrow is being rotated clockwise around the circle, in the sequence A-C-E, B-D-F etc. Therefore, figure D is the odd one out because the arrow has been reflected instead of rotated.

45. (C) We are counting triangles in an equilateral triangular grid with 4 rows. The standard formula is:

$$\text{Total triangles} = \left[\frac{n(n+2)(2n+1)}{8}\right]$$

Here, n = 4 (number of rows). Substituting

$$\text{Total triangles} = \frac{4(4+2)(2\times4+1)}{8}$$

$$= \frac{4\times6\times9}{8} = \frac{216}{8} = 27$$

CRITICAL THINKING

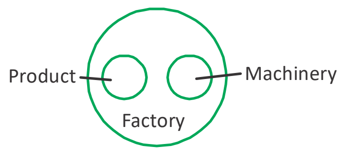
46. (B) The university cannot provide grants (Statement II), so the college decides to become autonomous. The students oppose this decision (Statement I) because they don't want the change.

Cause $\rightarrow$ Statement II

Effect $\rightarrow$ Statement I

47. (A) C $\rightarrow$ S (child) $\rightarrow$ U (grandchild). Since S is male, C must be S's parent. No info giving C's gender explicitly. But P is father in law of F implies F's spouse is child of P; that spouse is S? Wait F's children include N and U, whose father must be F's spouse. S is father of U and N $\rightarrow$ S is F's husband. Therefore S is husband of F. Then S is son of C, so C is parent of S and thus grandparent of U. Gender of C: since S is son of C, C could be mother or father. But options: Grandmother, Grandfather, Father, Mother. C is grandparent of U; male or female? Given typical naming, since P is father-in-law of F (P male) and P is father of F's spouse; spouse is S; so P is father of S. But we also have S is son of C, so C must be spouse of P (other parent of S). So C is mother of S (since P male father). Thus C is grandmother of U.

48. (D)



49. (D) Statement I alone gives number of rams but not how many are black. Statement II alone gives total black sheep (40% of 50 = 20) but not how many of those are female (ewes). Together still don't tell how many of the black sheep are ewes.

50. (A) The new math department chairperson is chosen based on seniority. From the information given, Ms. Wilson has more seniority than Ms. Brown but less than Mr. Tony, while Mr. Rohan is more senior than Ms. Wilson but less than Mr. Tony. Arranging them by seniority, we get: Ms. Brown < Ms. Wilson < Mr. Rohan < Mr. Tony. Since Mr. Tony does not want the position, the most senior eligible person is Mr. Rohan. Therefore, Mr. Rohan will be appointed as the new chairperson.